Question 1:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X Y | 1 | 2 | 3 | 4 | totals |
| 1 | 33 | 26 | 10 | 20 | 89 |
| 2 | 21 | 10 | 13 | 12 | 56 |
| 3 | 16 | 18 | 16 | 20 | 70 |
| totals | 70 | 54 | 39 | 52 | 215 |

(a)

f(X,Y) joint relative frequency table

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X Y | 1 | 2 | 3 | 4 | totals |
| 1 | 33/215=0.153 | 26/215=0.121 | 10/215=0.047 | 20/215=0.093 | 89/215=0.414 |
| 2 | 21/215=0.098 | 10/215=0.047 | 13/215=0.060 | 12/215=0.056 | 56/215=0.261 |
| 3 | 16/215=0.074 | 18/215=0.084 | 16/215=0.074 | 20/215=0.093 | 70/215=0.325 |
| totals | 70/215=0.325 | 54/215=0.252 | 39/215=0.181 | 52/215=0.242 | 215/215=1 |

(b)

fY|X(**·**|x) condition relative frequency for each possible value of x

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X Y | 1 | 2 | 3 | 4 | totals |
| 1 | 33/89=0.371 | 26/89=0.292 | 10/89=0.112 | 20/89=0.225 | 89/89=1 |
| 2 | 21/56=0.375 | 10/56=0.179 | 13/56=0.232 | 12/56=0.214 | 56/56=1 |
| 3 | 16/70=0.229 | 18/70=0.257 | 16/70=0.229 | 20/70=0.285 | 70/70=1 |
| totals | 70/215=0.325 | 54/215=0.252 | 39/215=0.181 | 52/215=0.242 | 215/215 |

(c)

Yes. Since fY(**·**|x) changes as X changes, X and Y are related over Ω (by definition)

The conditional mean of Y given X:

For X=1, E(Y|X=1) = y|x(Y|X=1) = (1)(0.371)+(2)(0.292)+(3)(0.112)=(4)(0.225) = 2.191

For X=2, E(Y|X=2) = y|x(Y|X=2) = (1)(0.375)+(2)(0.179)+(3)(0.232)=(4)(0.214) = 2.285

For X=3, E(Y|X=3) = y|x(Y|X=3) = (1)(0.229)+(2)(0.257)+(3)(0.229)=(4)(0.285) = 2.57

Hence, the form of the relationship between X and Y is: fY(**·**|x) increases as X rises.